Journal of Statistical Physics, Vol. 74, Nos. 3/4, 1994

The Dynamic Critical Exponent of the Three-Dimensional Ising Model

R. Matz,[1] D. L. Hunter,[1] and Naeem Jan[1]

Received August 26, 1993

We measure the dynamic exponent of the three-dimensional Ising model using a "damage spreading" Monte Carlo approach as described by MacIsaac and Jan. We simulate systems from $L=5$ to $L=60$ at the critical temperature, $T_c=4.5115$. We report a dynamic exponent, $z=2.35\pm0.05$, a value much larger than the "consensus" value of 2.02, whereas if we assume logarithmic corrections, we find that $z=2.05\pm0.05$.

KEY WORDS: Dynamic critical exponent; Ising model; damage spreading.

Reliable estimation of the dynamic critical exponents is still an outstanding problem. In fact recent works even suggest uncertainty in the value of the dynamic exponent for the two-dimensional Ising model. Poole and Jan[(1)] and Manna[(2)] report a value of z for this model of ~ 2.27, which is appreciably higher than the "consensus" value of 2.13. MacIsaac and Jan[(3)] with a different "damage spreading"[(4)] approach find for the two-dimensional system $z=2.16\pm0.02$, a value which is in good agreement with the "consensus" value of 2.13 and also with the recent slightly higher results of Ito[(5)] and Adler[(6)] (as cited by Ito), $z\approx 2.17$. We apply this method to the three-dimensional Ising model at its critical temperature, $J/kT_c = 0.221655$.[(7)]

A brief description of the method is now given, but the interested reader is referred to ref. 3 for details and justification of the method. A system, labeled A, of size $L*L*L$ is cloned to give a replica, labeled B. An extra layer of boundary spins is kept up in system A and down in system B. Both systems are allowed to equilibrate with heat-bath transition

[1] Physics Department, St. Francis Xavier University, Antigonish, Nova Scotia, Canada B2G 1C0.

0022-4715/94/0200-0903$07.00/0

probabilities.[8] Identical sites of both A and B lattices are visited at the same time, and the same random number is used to determine the status of the spin at this site. A Monte Carlo time step (MCS) consists of randomly visiting each site of the lattice once. After equilibration, the boundary layer is replaced by periodic boundary conditions. The set of sites in different states in a comparison of A and B is called the damage. The equilibrium damage with the pinned boundaries is the spontaneous magnetization and the Monte Carlo procedure with the periodic boundaries is continued until all the damage disappears, i.e., both systems A and B are in the same configuration. This procedure is repeated 5000 times for the smaller systems ($L \leqslant 15$) and at least 2000 times for the systems with $15 < L < 45$ and 500 times for $L = 50$ and 60. The total CPU time taken is approximately 90 days on an HP 9000-720 workstation. For further details of the Monte Carlo method see Binder[9] and on "damage spreading" see Coniglio *et al.*[10] and Glotzer *et al.*[11]

The time taken for the magnetization to disappear is a measure of the relaxation of the system from an initial equilibrium state to one which has lost all memory of the past. The average damage vs. time is shown in Fig. 1 for 200 trials and $L = 27$ in three dimensions and also for $L = 60$ in two dimensions. The average is obtained by dividing the damage by the total number of trials, *not* by the number of trials with nonzero damage at the time considered. This figure is shown to display the relaxation of the magnetization, or damage, and is not used to determine the numerical

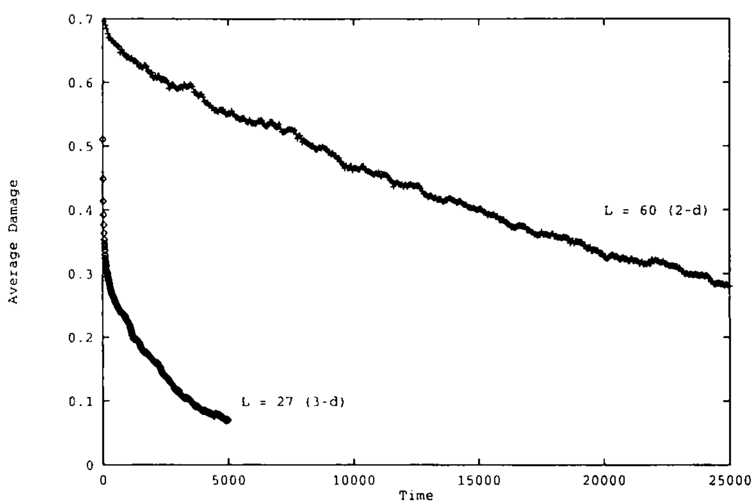

Fig. 1. The decay of the average damage, for $L = 27$ at T_c, with time in MCS for 200 trials. The decay of the damage for $L = 60$ in two dimensions at T_c is also shown.

results reported here. Note the rather sharp drop of the equilibrium damage as the pinned boundaries are replaced by periodic ones. This is followed by slower modes of relaxation. The average time taken for the damage to decay to zero is related to the slowest mode of relaxation τ, and dynamic scaling[12] asserts that

$$\tau \sim L^z \tag{1}$$

The average time for the magnetization to disappear is 265 MCS (Monte Carlo steps) for $L = 8$, 5990 MCS for $L = 30$, and 28300 MCS for $L = 60$. An attractive feature of this method is that there is no need to make any assumptions about the form of the relaxation curve, e.g., one- or two-exponential fits, etc., problems which plague some methods. We have an unambiguous method for measuring τ. Figure 2a shows a plot of these data on a log–log scale. The slope is 2.35 ± 0.05. This value is higher than the popular value of 2.02 ± 0.02 (ref. 13 and references therein), and also the more recent slightly higher estimates near 2.07.[5]

We are somewhat perplexed by this large value. The recent work of de Arcangelis *et al.*[14] used "damage spreading" from a single central site to measure the fractal dimensionality of the damage cloud as a function of L. They measured the mass of the instantaneous damage as a function of L and of R_g and observed that $d_f \approx 1.9$, not the value of 2.5 as expected. The data were further analyzed assuming various forms of logarithmic correc-

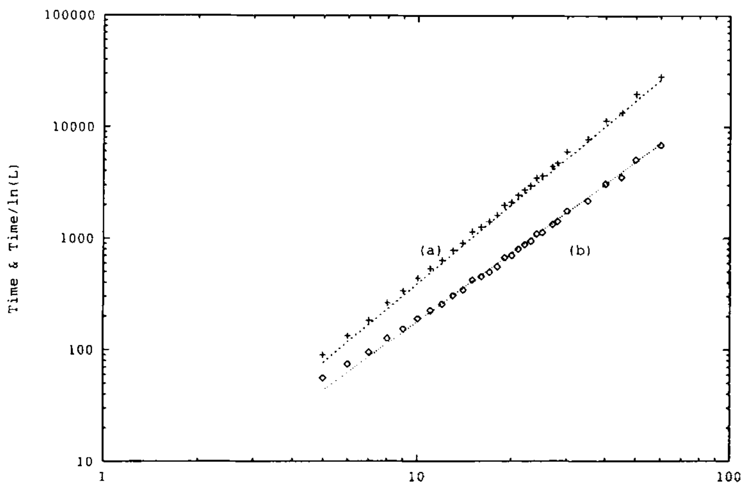

Fig. 2. (a) The average time taken for the damage to reach zero vs. system size. The slope of the straight line is 2.35. (b) The average time/ln(L) for the damage to reach zero vs. system size. The slope of the straight line is 2.05.

tions or logarithmic prefactors and only for these cases were they able to reconcile the effective fractal dimensionality with the expected asymptotic result. Implicit in their analysis is that there is also a log correction either associated with the time dependence of the "damage" mass or the time dependence of the spatial extent of the "damage" cloud.

We may view the relaxation of the damage as a process similar to the radioactive decay of atoms.[15] Thus the time-dependent decay of the damage is described by $D_0 e^{-t/\tau}$. If at time t_m the initial damage D_0, which scales with system size as L^{d_f}, decays to zero, then $D_0 e^{-t_m/\tau} \sim O(1)$. Now we find that t_m scales as

$$t_m \sim L^z d_f \ln(L) \tag{2}$$

The results assuming this form of scaling are shown in Fig. 2b. The straight line is one of slope 2.05 ± 0.04.

Figure 3 is a semilog plot of the number of trials with nonzero damage as a function of time for 2D systems (the curves that extend to 15000 MCS) and 3D systems (the shorter curves). Here there is clear indication that there is an exponential decay in the number of survivors. Figure 4 shows the average damage per spin as a function of time, where the average is taken from the number of *surviving* trials at that time. There is clear evidence of finite-size effect in that the damage is $\sim L^{-\beta/\nu}$ and stays that

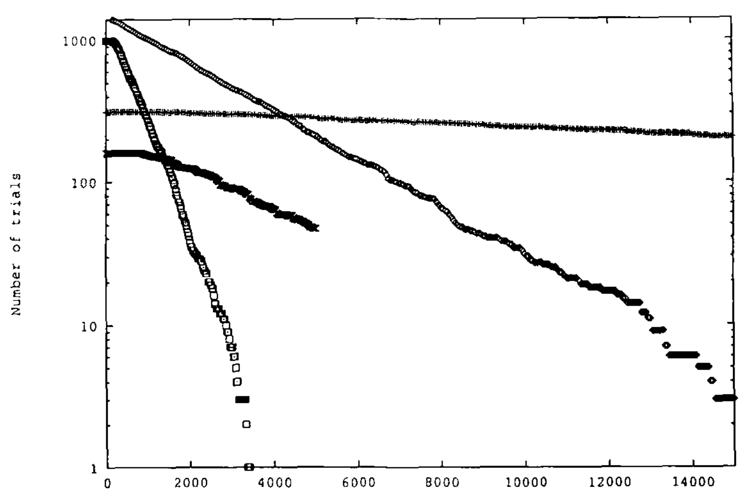

Fig. 3. The log of the number of surviving trials vs. time for 2D systems (upper curves) and 3D systems (lower curves). (+) $L=60$ system, ($\Diamond$) $L=20$ (2D), ($\times$) $L=27$, ($\Box$) $L=13$ (3D).

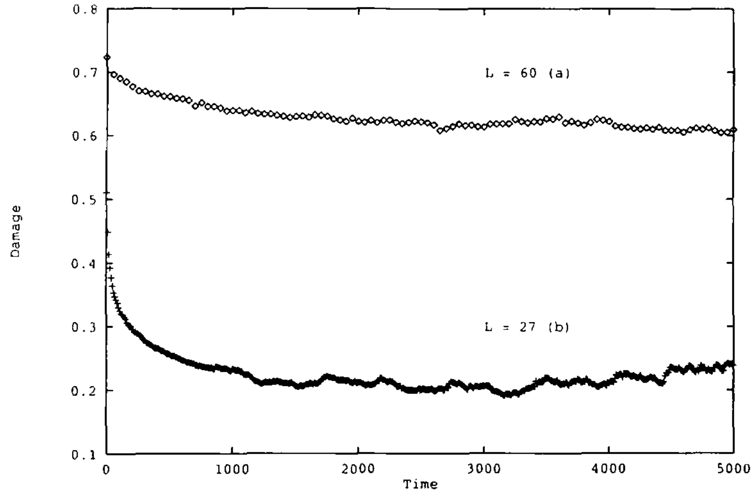

Fig. 4. The average damage vs. time for (a) $L = 60$ (2D) and (b) $L = 27$ (3D) systems where the average is taken from those trials with nonzero damage at that time step. The steady-state damage is $\sim L^{-\beta/\nu}$.

way until a fluctuation drives it to zero. This is very similar to the observed behavior of the equilibrium magnetization of finite systems; the magnetization at T_c oscillates between positive and negative values of $L^{-\beta/\nu}$ and the time taken to tunnel from one value to another is a measure of τ, the characteristic relaxation time. These results are at odds with logarithmic corrections.

We have used a method which in principle is unambiguous in its implementation with no subjective assessment of the relaxation time τ. The method was applied to the two-dimensional Ising model and led to a result for the dynamic exponent z which is in remarkable good agreement with the much larger systems investigated in ref. 5. The same method applied to the three-dimensional Ising model leads to an unexpected large value (2.35 ± 0.05) for z. Assuming the presence of a logarithmic correction which is implicit in ref. 14, we find that z is approximately 2.05 (see also the recent work of Hunter *et al.*[16]). This value is in keeping with earlier work, the recent work of Stauffer[15] and Ito[5] referred to above, and Heuer.[17] However, if we assume that a logarithmic correction is also appropriate for the two-dimensional Ising system,[3] then we obtain a value of z of $\leqslant 2.0$, much lower than the expected value of 2.17. We have found no direct evidence to justify logarithmic corrections.

ACKNOWLEDGMENTS

N.J. and D.L.H. are supported by NSERC grants. We thank A. Coniglio, P. Poole, S. Glotzer, D. Stauffer, and K. MacIsaac for discussions.

NOTE ADDED IN PROOF

D. Stauffer has obtained results which are roughly compatible with those reported here, using an independent computer program.

REFERENCES

1. P. H. Poole and N. Jan, *J. Phys. A: Math. Gen.* **23**:L453 (1990).
2. S. S. Manna, *J. Phys.* (Paris) **51**:1261 (1990).
3. K. MacIsaac and N. Jan, *J. Phys. A: Math. Gen.* **25**:3585 (1992).
4. M. Creutz, *Ann. Phys.* (N.Y.) **167**:62 (1986).
5. N. Ito, *Physica A* **192**:604 (1993); **196**:591 (1993); and references therein.
6. J. Adler, private communication; B. D. Dammann and J. D. Reger, *Europhys. Lett.* **21**:157 (1992).
7. J. Adler, *J. Phys. A: Math. Gen.* **16**:3585 (1983).
8. B. Derrida and G. Weisbuch, *Europhys. Lett.* **4**:657 (1987).
9. K. Binder, *Monte Carlo Methods in Statistical Physics*, Vol. 7 (Springer, Heidelberg, 1984).
10. A. Coniglio, L. de Arcangelis, H. J. Herrmann, and N. Jan, *Europhys. Lett.* **8**:315 (1989).
11. S. Glotzer, P. H. Poole, and N. Jan, *J. Stat. Phys.* **68**:895 (1992).
12. P. C. Hohenberg and B. I. Halperin, *Rev. Mod. Phys.* **49**:435 (1977).
13. S. Wansleben and D. P. Landau, *Phys. Rev. B* **43**:6006 (1991).
14. L. de Arcangelis, H. J. Herrmann, and A. Coniglio, *J. Phys. A: Math. Gen.* **23**:L265 (1990).
15. D. Stauffer, *J. Phys. A* **26**:L599 (1993).
16. D. L. Hunter, L. de Arcangelis, R. Matz, P. H. Poole, and N. Jan, *Physica A* **196**:188 (1993).
17. H. O. Heuer, *J. Phys. A: Math. Gen.* **25**:L567 (1992).

Communicated by D. Stauffer